University of Saska

Department of Mathematics & Statistics

Time: 3 hours MATH 224.3 Final Examination April 18, 2002

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- Encode your student number and PRINT your name on the opscan sheet.
- CLOSED BOOK - NO CALCULATORS ALLOWED
- TWO SHEETS OF FORMULA ALLOWED
- 22 questions: Questions 1–17 are worth 5 marks each.
  18–22 are worth 3 marks each.
  Total: 100 Marks.

Answer choices for questions 1–14

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

(F) 5 (G) 6 (H) 7 (I) 8 (J) 9

1. If $y = y(x)$ is the solution to the IVP $\dfrac{dy}{dx} = \dfrac{2y}{x^2+2x}$; $y(1) = 3$, then $y(4) =$

2. If $y = y(x)$ is the solution to the IVP $\dfrac{dy}{dx} - \dfrac{3y}{x} = x^4$; $y(1) = -\dfrac{1}{2}$, then $y(2) =$

3. If $y = y(x)$ is the solution to the IVP $y'' + 2y' + y = 0$; $y(0) = 3$; $y'(0) = -3$, then $y\big(\ln(\frac{1}{3})\big) =$

4. If $y = y(x)$ is the solution to the IVP $[2 + \sin(y)]dx + [(x - y)\cos(y) - \sin(y) - 2]dy = 0$; $y(1) = 1$, then $y(5) =$

5. If $y = y(x)$ is the solution to the IVP $\dfrac{dy}{dx} = \dfrac{x+2y}{x}$; $y(1) = -\dfrac{3}{4}$, then $y(6) =$

6. If $y = y(x)$ is the solution to the IVP $y'' + 5y' + 6y = 36x$; $y(0) = -5$; $y'(0) = 6$, then $y(2) =$

.../2

7. If $y = \sum_{n=0}^{\infty} c_n x^n$ is a power series solution to the IVP $xy' - 3y = 2x - 6;\ y(1) = 3,$ then $c_3 =$

8. For the sequence $a_n = \left(\frac{n-7}{2n+4}\right)^{\frac{3n-2}{1-n}}$ $\lim_{n\to\infty} a_n =$

9. For the sequence $a_n = \frac{(2n)^{2n}}{[(n+4)!]^2}$, the limit of the ratio of successive terms $\lim_{n\to\infty} \frac{a_{n+1}}{a_n}$ exists and is equal to ae^b where $a + b =$

10. In the Maclaurin series for $f(x) = \frac{x^4(4-x)^{3/2}}{3}$, the coefficient of x^6 is equal to 2^{-a} where $a =$

11. The sum of the series $\sum_{n=1}^{\infty} \frac{3^n - 2^{n+1}}{4^{n-1}}$ is

12. The sum of the series $\sum_{n=5}^{\infty} \frac{24}{n^2 - 2n}$ is

13. The sum of the series $\sum_{n=1}^{\infty} (-1)^{n-1} 5 \left(\frac{1}{4}\right)^n$ is

14. In the Maclaurin series for $\frac{d}{dx}\left(5x^2 \arctan(x)\right)$, the coefficient of x^6 is

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Answer choices for questions 15–17

(A) only converges for $x = 0$ (B) $(-\infty, \infty)$ (C) $(-1, 1)$ (D) $(-1, 1]$

(E) $[-1, 1)$ (F) $[-1, 1]$ (G) $(-e, e)$ (H) $(-e, e]$ (I) $[-e, e)$ (J) $[-e, e]$

15. Find the interval of convergence of the power series $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n+1} x^n$.

16. Find the interval of convergence of the power series $\sum_{n=2}^{\infty} (-1)^n \frac{e}{\sqrt{n}\ln(n)} x^n$.

17. Find the interval of convergence of the power series $\sum_{n=1}^{\infty} (-1)^n \frac{10^{500n}}{n!} \left(1 + \frac{1}{n}\right)^{-n} x^n$.

Answer choices for questions 18–22

A. Converges absolutely

B. Converges conditionally

C. Diverges to ∞

D. Diverges to $-\infty$

E. Diverges (not to $\pm\infty$)

Choose the answer choice which best describes the indicated series.

18. $\sum (-1)^n \frac{n^2}{n^3+1}$ 19. $\sum (-1)^{n+1} \cos(n\pi)$ 20. $\sum (-1)^n \frac{n!}{n^n}$

21. $\sum (-1)^n \left(\frac{n}{2n+1}\right)^n$ 22. $\sum \left(\frac{2^{1000n}}{n!} - \frac{1}{1000n}\right)$

** The End **